Planning of a supply chain for anti-personal landmine disposal by means of robots

Rafael Guillermo García-Cáceres
Doctor en Ingeniería de la Universidad de los Andes. Profesor asociado de la Decanatura de Ingeniería Industrial, Escuela Colombiana de Ingeniería Julio Garavito, Colombia.
Correo electrónico: rafael.garcia@escuelaing.edu.co

Julián Arturo Aráoz-Durand
Doctor en Computer Science de la University of Waterloo. Profesor visitante del Departamento de Matemáticas e Investigación de Operaciones, Universidad Politécnica de Cataluña, España. Profesor Titular (retirado) Universidad Simón Bolívar, Venezuela.
Correo electrónico: julian.araoz@upc.es

Fernando Palacios-Gómez
Doctor en Operations Research de la Texas University at Austin. Profesor titular (retirado), Departamento de Ingeniería Industrial, Universidad de los Andes, Colombia.
Correo electrónico: fpalacio@uniandes.edu.co

ABSTRACT: The current paper presents a Mixed-Integer-Linear Programming Model (MIP) which incorporates strategic and tactical management decisions into the supply chain of an anti-personal landmine robotic detection and disposal system. Originally based on a mixed-integer-non-linear programming model (MINLP) with stochastic elements, of which it is an approximation, the MIP model is obtained by means of two solution procedures that include redefining variables, treating stochastic and non-linear constraints, and incorporating valid constraints. The model included considerations such as uncertain procurement, stochastic inventories in plants, production scales, supply-production-distribution capacities, particular distribution-production infrastructure, location-allocation considerations, stochastic demand, and BOM. Additionally, the models detail optimal helicopter operation by considering each period’s trip frequency during the planning horizon. Finally, a sensibility analysis of the way in which parameters variations affect overall costs is presented. The suggested solution procedure is considered satisfactory in terms of time for the analyzed example.

KEYWORDS: mathematical programming applications, integer programming, non-linear programming, stochastic programming, logistics, supply chain, robots, anti-personal landmines

1. Introduction

The UN ban on the use of anti-personal landmines as war material (justified by their devastating impact on the world population, specially civilians who are not involved in the conflicts) materialised in the 1997 Ottawa Convention, which endeavoured to find an unanimous agreement on the establishment of observation centres for the surveillance over the de-mining of contaminated territories, the promotion of necessary measures for their disposal, the ban on their production and use as well as the destruction of all existing arsenals (Landmine Monitor Report 2009).

Mine-laying is a common practice in war conflicts due to the low cost and simplicity of construction of anti-personal mines (at least in low sophisticated devices).
In army operations, mines are used in a variety of ways: to block the advance of the enemy into specific areas, or to lead them into other ones where they can be more effectively attacked; to obstruct their movements during attacks; to prevent them from using resources in areas that will be abandoned (natural resources, facilities, equipment, communication routes, etc.); to reinforce natural or artificial obstacles; to prevent enemy retreat or to facilitate one's own, and to get in the way of the enemy's logistic support.

Mines have meant another issue in the horror of war because of the damage they cause to civilians. The main reasons are, on one hand, that landmine laying and clearance dynamics is hardly predictable and carelessly registered, and therefore impossible to analyse. On the other hand, partial mine-clearing of fields, also known in the military jargon as 'gap opening', is a general practice.

The consequences of mine explosions are burns, multiple wounds and infections caused by splinters. Wounds may have deadly consequences due to the impact of the explosion itself or the evolution of the injuries received. Besides, there is constant fear amongst the affected population because of the risk of injury they are permanently exposed to.

The 2003 annual mine report informed that, by 2002, anti-personal mines had already injured 300.000 people around the world. The number of mutilated people in Angola is said to be around 20.000 according to the United Nations, and 70.000 according to “Doctors without Borders”. For a 10 million people population, the rate could go from 1 out of 500 inhabitants, to 1 out of 145. In Somalia, the approximate rate is 1 out of 650, and in Cambodia, 1 out of 234. In Colombia, the number increased from 216 injured inhabitants in 2001 to 530 in 2002, between January and April of 2003, 151 and 1000 casualties and wounded people were reported in average and the following years. In addition, the 2009 annual mine report estimated the number of casualties in Colombia caused by mines, explosive remnants of war and victim-activated improvised explosive devices, to be 6.696 between 1999 and 2008.

The Red Cross International Committee, which has produced and delivered 7.402 prostheses and 311 orthoses for mine victims in 2011 (ICRC, 2011), estimated by 2003 that 800 people are killed by mines every month, 26 persons a day (ICBL, 2003), while figures from the U.S. Department of State talk about 26.000 casualties and wounded people every year (72 per day). According to estimations published in IDOC Internazionale (Dentico, 1995), for each victim surviving to a mine explosion, two people have died. In some countries, 75% of the survivors require amputations.

Figures are difficult to calculate since most highly-mined countries (with a recently ended conflict or still in conflict) lack the necessary infrastructure to transport and look after the victims on time.

According to “Physicians for Human Rights” (Human Rights Watch/Arms Project and Physicians for Human Rights, 1993), a full recovering medical treatment costs between US$3.000 and US$5.000, while the equipment needed by a child victim to walk again is priced above US$3.000.

The traditional landmine detection procedure on a land strip that has been classified as contaminated by mines is first carried out remotely by causing explosions, either by grenades dropped from aircraft or helicopters, by impacts of artillery shells, or with special vehicles that detonate landmines by direct contact. In a second step, specialized personnel are sent in order to detect remaining mines with hand held devices. Given the many types of mines and the mechanisms used to hide them, it becomes clear that this procedure is far from being efficient, cost saving, safe or environmentally sustainable.

As a result of advances in robot technology, the latter has emerged as a viable innovative option for landmine detection and disposal. Although these developments do not allow large scale applications, it becomes clear that this procedure will enhance safety for specialised personnel, and is much more environmentally friendly when compared to other procedures currently in use.

Considering the future viability of this option, a mathematical programming model to design supply chains for landmine disposal might probably become a sine qua non. The model would have to encompass technical robot requirements, as well as relevant production and logistic considerations. Such is the scope of the current paper.

2. Background

The following review focuses on some topics generally considered as relevant for strategic and tactical decision making in supply chains that have additionally been treated by means of mathematical programming models.

Among the first literature reviews on the topic, the one published by Aikens (1985) makes a description of the most relevant aspects of supply chain modelling for single echelon systems with deterministic demand. The fundamental aspects were associated to facilities location in plants and distribution centres (DCs), as well as raw material and final
product distribution. The problem size was then limited by the absence of a computationally adequate MIP optimiser.

The evolution of supply chain research has spread out in several directions since then. In the first place, as properly indicated by the above mentioned author, towards dynamic modelling considerations and handling of inventories associated with one agent (Cohen and Lee, 1988); in the second place, looking forward to some greater satisfaction of the final consumer (Arntzen et al., 1995); in the third place, towards the development of distribution systems (Geoffrion and Powers, 1995); and last, seeking for a better co-ordination of logistic operations in different stages of the supply chain (procurement, production and distribution) (Thomas and Griffin, 1996). The aspects considered by these authors included: lead times, procurement and distribution channel capacity, scale economies and bill of materials, among others.

Towards the end of the twentieth century, due to increased pressures caused by the internationalisation of economy, and taking in new developments in computational processing, the characterization of the global supply chain concept was consolidated by Vidal and Goetschalckx (1997). In order to solve these new aspects, optimisation and heuristic procedures supported by commercial software were developed by Goetschalckx et al. (2002), resulting in satisfactory practical results (Vidal and Goetschalckx, 1997). Some study topics on supply chains have included uncertainty in the procurement (Gupta and Maranas, 2003; Chen and Lee, 2004). Dong et al. (2004) developed a model oriented towards the equilibrium of the supply chain when there is stochastic demand from retailers to manufacturing plants. The model considered a number of manufacturing plants that produce homogeneous articles, which are later bought by the retailers. The objective consisted on maximizing the total profit. Eskigun et al. (2005) proposed the design of a Supply Chains distribution network, considering delivery times, and locations and vehicles capacities. The study by Agarwal et al. (2007) uses an interpretative structural model that allows analyzing the way in which the relationships between the variables that make up the supply chain, affect its agility and flexibility. An architecture that allows the assurance, simplification and synchronization of simulation models within a supply chain, is proposed in Iannone et al. (2007). Actual and expected delivery times are considered within the problems objective. Finally, Sawik (2009) achieved the integration of material scheduling, material supply and product assembly using a mixed integer programming approach product.
Towards the end of the twentieth century the increased pressure by the economic internationalization of supply chain, new global aspects had to be considered these included: transfer prices (Vidal and Goetschalckx, 2001) and particular topics on design of operations (Mula et al., 2011). Recently García et al. (2011) presented a work on domestic supply chain environments that included new aspects like reliability, scale economies, organizational considerations, uncertain demand, among other relevant aspects.

Such is the topic of the current document, which boasts an incorporation of strategic and tactical aspects in both static and dynamic contexts, including particular infrastructure and logistics considerations about distribution and production, together with some other important items like reliability of procurement channels, factory location, and stochastic demand. Summarising, the paper presents a model, a solution procedure and a sensibility analysis, applied to a particular example. In order to explain the model, each of the aspects it deals with is presented in both conceptual and mathematical contexts.

3. The Model

Although more complex, supply chain models integrating strategic and tactical decisions are known to allow closer approximations to reality than those only linked to either type of decision (Goetschalckx et al., 2002). In fact, the medium scale optimisation instances found in the realm of this particular case make it advisable to apply the former model type. In order to introduce the MINLP and MIP models, a consistent notation is presented as the different aspects of the supply chain are introduced.

Aspects taken into account

The different aspects considered in the model are treated in detail in the following sections.

3.1. Procurement reliability

A very important consideration in supply chain strategic management has to do with channel selection, not only because it involves raw material and supply costs, but procurement lead times and raw material quality too, which all come up as reliability requirements in the production stage. Special attention was given in this paper to the proposals of Vidal and Goetschalckx (2001), which used binary variables to model procurement channel reliability constraints, and then applied them as relevant choosing criteria.

Sets

\[ A_I(i,p) : \text{raw material used up by supplier } i \text{ in product } p \]

\[ U_I(j) : \text{plant } j \text{ suppliers} \]

\[ J : \text{plant locations} \]

\[ P : \text{robot types} \]

\[ PA(j) : \text{robot types made in factory } j \]

Parameters

\[ PROB_{ij}^a = \text{reliability of the supply channel that links supplier } i \text{ with plant } j \text{ through raw material } a \text{ (percentage)}. \]

\[ PRO_{ij}^p = \text{reliability goal in the production of robot type } p \text{ at plant } j \text{ (percentage)} \]

Variables

\[ v_{ij}^a = \begin{cases} 1 & \text{if supplier } i \text{ provides plant } j \text{ with raw material } a \\ 0 & \text{otherwise} \end{cases} \]

Expression for supply channel choice reliability:

\[
\prod_{i \in U_I(j)} \left( \prod_{a \in A_I(i,p)} \left( PROB_{ij}^a \right)^{v_{ij}^a} \right) \geq PRO_{ij}^p, j \in J, p \in PA(j)
\]

Reliability is modelled through the supply compliance percentage required by the factories for each raw material, which can be obtained by means of a feasible combination of their suppliers’ reliability percentage measurements.

3.2. Bill of Materials (BOM)

The bill of materials constraint has a twofold function in the model, both linking the procurement and distribution stages (between plants and demand zones (DZs)), and quantifying the raw material amounts required to satisfy the demand. Given the importance of these two aspects, this constraint becomes particularly necessary when choosing suppliers and procurement channels in the planning horizon.

Sets

\[ A : \text{raw material} \]

\[ PA(a) : \text{robot types using raw material type } a \]

\[ RIJ : \text{supply network made up of logistic links between suppliers and plants} \]

\[ RJK : \text{supply network composed of logistic links between plants and DCs} \]

Parameters

\[ q_{ap} = \text{amount of type } a \text{ raw material used for the construction of one type } p \text{ robot (resource units / robot)} \]
3.3. Allocation of DCs regarding one singular supply source

The geographical location of production plants and distribution centres depends on the particular operational conditions of the army. A national army generally consists of divisions that are themselves composed of brigades, which are in turn subdivided into battalions that can have assigned special tasks like engineering, cavalry, artillery, infantry, special forces, etc. Yet, the division commanding tasks are assigned to higher level battalions, namely the division ones. To avoid overlapping of competencies, each brigade has its assigned territory. In this manner, the security and efficiency of army operations and the monopoly of war material are safeguarded.

Consequently, any decision making endeavours to outline the location of production plants within division battalions, and their associated DCs in brigade battalions. Additionally, for army operation security reasons, each DC is responsible for the de-mining of just one DZ. In modelling this aspect, Geoffrion and Grave’s [16] proposal has been used as a referential milestone, as far as it links the aggregate demand of each DZ to a single procurement source. This constraint is particularly relevant due to the aforementioned army hierarchy. The correspondent expressions are shown below:

\[ \sum_{(i,j)\in P_{i}} s_{ij} = \sum_{p\in P_{i}} \sum_{(j,k)\in R_{jk}} q_{ijp} x_{jk} \quad j \in J, a \in A. \quad (3.2) \]

3.4. Integration of production, distribution and allocation stages

The construction of a robot is typically modular, and it is foreseeable that the main production activities (assembly or manufacture) are carried out in factories, whereas the repairing activities are restricted to distribution centres (DCs). However, the constraint presented here is not only to aim these simple tasks, but also the production of new (or innovative) robot components. Additionally, in the tactical aspect, the model defines robot production and distribution for each planning period.

In proposing these constraints we seek to establish a cross link between the strategic and tactical decisions of the chain. In regard to the former ones, the average aggregate distribution from plants and DCs is linked to the production periods in the factories, in order to correctly choose their relative location and assignment. Division and brigade battalions are usually lodged in strongly guarded locations and have various means of transportation among which their habitually well maintained roads are the most common.

\[ P_{j}(j,k) : \text{robot types sent from factory } j \text{ to DZ } k \]
\[ P_{k}(k) : \text{robot types sent to DZ } k \]
\[ T : \text{periods} \]

\[ N = \text{number of periods.} \]

\[ d_{i}^{p} = \text{average periodic amount of type } p \text{ robots used at DZ } k \text{ in the planning horizon (units/period)} \]
\[ y_{i}^{p} = \text{amount of type } p \text{ robots delivered at DZ } k \text{ during period } t \text{ (units/period)} \]

Expression for unique permissible production and distribution source choice for a DC:

\[ x_{jk}^{p} = d_{jk}^{p} \quad (j,k) \in R_{jk}, \quad p \in P_{j}(j,k) \quad (3.4) \]

Expression for link between both average periodical production and periodical production, delivered at a DZ:

\[ \sum_{(j,k) \in R_{jk}} y_{jk}^{p} = N d_{k}^{p} \quad k \in K, \quad p \in P_{k}(k) \quad (3.5) \]

Note that both sides of the balance equation show the aggregated amount of robots at DZs.
3.5. Selection of procurement channels and location of factories

Procurement and location are core decisions in logistics, due to their associated strategic and tactical potential costs along the entire planning horizon. In addition, each supplier bases his calculations on a minimum offer, which depends on his procurement policies and on a maximum supply bound which in turn relates to his production capacity. Consequently, the suggested model includes throughput capacity constraints for each supply channel and production plant.

**Sets**

\(AIJ(i,j)\) : raw material types provided by supplier \(i\) at plant \(j\)

\(JI(i)\) : plants provided by supplier \(i\)

\(KJ(j)\) : DZs supplied by factory \(j\)

**Parameters**

\(CAP_p^j\) = periodic production capacity of plant \(j\) for manufacturing type \(p\) robot, in the planning horizon (units / period)

\(O^p\) = capacity fraction used in the production of type \(p\) robots (%)

\(SMAX_i^a\) = maximum periodic amount of type \(a\) raw material provided by supplier \(i\) in the planning horizon (units of raw material / period)

\(SMIN_i^a\) = minimum periodic amount of type \(a\) raw material provided by supplier \(i\) in the planning horizon (units of raw material / period)

**Variables**

\(a_j = \begin{cases} 1 & \text{if the plant is located in } j \\ 0 & \text{otherwise} \end{cases}\)

Logical expression linking suppliers to plants, which becomes necessary because a procurement channel can only be selected if its supplied plant location is selected too.

\(v_{ij}^a \leq a_j \quad j \in J, i \in J(I(j)), a \in AIJ(i,j)\) (3.6)

Expressions for procurement capacity:

\(SMIN_i^a v_{ij}^a \leq S_j^a \leq SMAX_i^a v_{ij}^a \quad i \in I, j \in J(I(i)), a \in AIJ(i,j)\) (3.7)

Raw material flow from suppliers to factories depends on procurement channel reliability. The left (right) side of the above constraint allows to model minimum (maximum) procurement conditions, traditionally imposed by some suppliers, deriving from their production and logistic inflow capacity.

Expressions for production capacity:

\[
\sum_{k \in K(j)} O^p v_{kt}^p \leq CAP_p^j a_j \quad j \in J, t \in T, p \in PJ(j)
\] (3.8)

Robot production is bounded by factory capacity, which can be feasible, and consequently positive, only if factory location is also feasible.

3.6. Scale economies

Operation levels define production and distribution (from plants to DZs) scale economies in each time period, a relation that can be modelled through mathematical programming, by defining the production and distribution range sizes for which a related differential cost has been established. The average unitary cost decreases gradually along ranges, up to the point where the capacity is saturated. From then on, an increase can be observed as the demand grows beyond the available capacity and has to be satisfied either by outsourcing or reinvestment. Scale economies have been considered in this article because of their relevance to the design of the supply chain.

**Sets**

\(E(p)\) : type \(p\) robot production scales

**Parameters**

\(GMAX_k^p e\) = maximum production bound for type \(p\) robots delivered at DZ \(k\) in operation scale \(e\) (robot units / period)

\(GMIN_k^p e\) = minimum production bound for type \(p\) robots delivered at DZ \(k\) in operation scale \(e\) (robot units / time period)

**Variables**

\(y_{kt}^p e\) = type \(p\) robots delivered at DZ \(k\) in operation scale \(e\) during period \(t\) (units / period)

\(w_{kt}^p e\) = \(\begin{cases} 1 & \text{if product } p \text{ is produced in operation scale } e \text{ during period } t \\ 0 & \text{otherwise} \end{cases}\)

Expression for operation scale: the number of robots distributed during a given time period must have been produced in any of such period’s production scales.

\[
y_{kt}^p = \sum_{e \in E(p)} y_{kt}^p e \quad k \in K, t \in T, p \in PK(k)
\] (3.9)

Logical expression for operation scales: one only robot production scale at the most, can be activated in a given time period.

\[
\sum_{e \in E(p)} w_{kt}^p e \leq 1 \quad t \in T, p \in PK(k)
\] (3.10)
Expression for operation scale bounds: in order to match periodical robot production to its corresponding scale, the following constraint is used:

\[
(G_{MIN_t}^p)w_t^p \leq (G_{MAX_t}^p)w_t^p, t \in T, p \in PK(k), e \in E(p)
\] (3.11)

Such constraint allows defining the operation scale at which the production of each robot type is carried out, framing it within its two corresponding bounds. Additionally, and together with constraints (3.9) and (3.10), it assures that the number of robots delivered from each plant to its associated DZs comes from only one production scale.

### 3.7. Distribution infrastructure

Distribution activities inside the demand zones (from brigades to mine contaminated areas) are usually hampered by lack or bad condition of access roads and by proneness to assaults by enemy forces. Consequently, the access is usually carried out using helicopters. Those aspects of the supply chain that are related to delivery into DZs are very important due to expensiveness of helicopter fleet operation and buying cost. The helicopter fleet size can be obtained by means of the following expression:

\[
\text{VARIABLES}
\]

\[
h_{kt} = \text{number of helicopters used at DZ } k \text{ during period } t \quad (\text{units / time period})
\]

\[
h_{max}^k = \text{minimum number of helicopters used at DZ } k \text{ in the planning horizon (units)}
\]

Expression for helicopter fleet minimum size

\[
h_{max}^k \geq h_{kt}, \quad k \in K, \ t \in T
\] (3.12)

The helicopter fleet minimum size at each DZ corresponds to the maximum number of helicopters used during a given time period within the studied planning horizon.

### 3.8. Some other considerations about supply chain capacity

As it has been shown, in studying the previous aspects, certain relevant capacity constraints were added to the supply chain model. Nevertheless, it is necessary to include some additional capacity considerations. Two relevant constraints are respectively associated to throughput production capacity due to raw material procurement bounds, and to the necessary infrastructure to carry out the distribution process from plants to DCs and within the DZs.

### Sets

\[
\text{PIA}(i,a) : \text{robots types that use raw material type } a \text{ provided by supplier } i
\]

### Parameters

\[
\text{HE}_k = \text{number of available helicopters in DZ } k
\]

\[
X_{MAX}^p = \text{distribution capacity for type } p \text{ robot from plant } j \text{ to DZ } k \text{ in the planning horizon (units / period)}
\]

\[
Z_{MAX}^k = \text{maximum number of feasible helicopter trips in DZ } k
\]

\[
\beta^p = \text{helicopter pay load capacity for transporting type } p \text{ robots (robot units / helicopter)}
\]

### Variables

\[
z_{kt} = \text{number of helicopter trips at DZ } k \text{ during period } t \text{ (trips / period)}
\]

Expression for distribution capacity bounds, due to supply channel capacity:

\[
SMAX_{IA}^a = \sum_{j \in P \cap K} \sum_{t \in T} \sum_{a \in A} y_{jtai}^p \cdot SMAX_{IA}^a, i \in I, j \in J, t \in T, a \in A(i, j)
\] (3.13)

The raw material used for a certain product must be adjusted within the raw material procurement bounds established by each supplier.

Expression for distribution capacity from plants to DCs:

\[
y_{jtai}^p \leq X_{MAX}^p, \quad j \in J, k \in K, \ t \in T, \ p \in P \cap K(j, k)
\] (3.14)

The above constraint takes into account the distribution capacity between plants and DCs.

Expression for distribution capacity within DZs:

\[
y_{jtai}^p \leq \beta^p \cdot z_{kt} 
\]

The above constraint contemplates both payload capacity and operational frequency of helicopter trips within DZs.

Expression for maximum number (maximum bound) of helicopters within DZs:

\[
h_{kt} \leq \text{HE}_k, \quad k \in K, \ t \in T
\] (3.16)

Expression for maximum number (maximum bound) of helicopter trips per time period within DZs:

\[
z_{kt} \leq Z_{MAX}^k, \quad k \in K, \ t \in T
\] (3.17)

### 3.9. Demand

Data collected by the local observatory will allow the classification of an area as contaminated by mines. The
consequent demand for robots is not determined by the number of mines to be detected, but the extension of the contaminated area, due to the fact that if a specific land strip is contaminated, then it has to be entirely scanned by robots, in disregard of how many mines it might have.

As for the robots, it has to be taken into account that a percentage of them will periodically be rendered useless. Additionally, when considering a DZ, the model assumes a stochastic contaminated area which is scanned by robots, where each robot type has an average scanning speed defined for each period. In order to suitably model the stochastic area, chance constraints are included (Charnes and Cooper, 1959); but this requires a certain level of compliance probability for each given constraint. The associated constraint (eq 3.16) is shown below:

### Parameters

\( \alpha \) = significance level

\( \theta_p \) = type \( p \) robot expected scanning speed in a given planning period (scanned area units / robot)

\( \varphi_p \) = expected contingent fraction of type \( p \) robots in a given planning period (percentage / period)

\( \pi_p \) = type \( p \) robot estimated operational life span (time units)

\( \Theta_k(\xi) \) = mine contaminated stochastic area in DZ \( k \) in the planning horizon (area units)

Chance constraint for DZ stochastic area robotic de-mining in the planning horizon

\[
\left( \sum_{\xi} \left[ \sum_{\theta} \left[ \sum_{\xi} \left[ \sum_{\varphi} \left[ \sum_{\pi} \left[ \sum_{\Theta} \left( 1 \right) \right] \right] \right] \right] \right] \right)_{\xi} \leq 1, \quad \forall \xi, \forall k
\]

(3.18)

The above constraints express the minimum permissible target probability required to scan the mine contaminated area in the planning horizon.

### 3.10. Variable bounds

\( s_{ij}^a \geq 0 \quad i \in I, \quad j \in J, \quad a \in A \)

\( x_{jk}^p \geq 0 \quad j \in J, \quad k \in K, \quad p \in PK(k) \)

\( d_k^p \geq 0 \quad k \in K, \quad p \in PK(k) \)

\( y_{ik} \geq 0 \quad k \in K, \quad t \in T, \quad p \in PK(k) \) and integer

\( y_{pe} \geq 0 \quad k \in K, \quad t \in T, \quad p \in PK(k), \quad e \in E(p) \) and integer

\( z_{kt} \geq 0 \quad k \in K, \quad t \in T \) and integer

\( h_k \geq 0 \quad k \in K, \quad t \in T \) and integer

\( h_{\text{max}} \geq 0 \quad k \in K, \quad t \in T \) and integer

(3.19)

### 3.11. Objective function

The planning model has to take into account the fixed costs of a series of supply chain activities, like construction (installation) and operation of factories and DCs, or logistic flows of raw materials and manufactured products between suppliers, plants, and DZs. In sum, the fixed costs of the procurement, production and distribution stages. Additionally, the model not only considers certain fixed costs (e.g., infrastructure) that can be estimated by assigning them a constant value along the entire planning horizon, but also those that go through periodical changes (like production and distribution), and are equally included in the planning horizon. Finally, the model follows Silver and Peterson’s proposal (1985) of inventory quantification, which includes safety stock factors, lead times, and inventory cycle factors. The proposal assumes that stochastic demand and deterministic lead times are independent for each raw material.

### Parameters

\( F_{ij}^a \) = type \( a \) raw material inter arrival time from supplier \( i \) to plant \( j \) (time units / planning horizon)

\( F_{jk}^p \) = type \( p \) robot inter arrival time between plant \( j \) and DZ \( k \) (time units / planning horizon)

\( FCI \) = inventory cycle factor (percentage)

\( FIS_{aj} \) = safety stock factor for type \( a \) raw material at plant \( j \) (time units / planning horizon)

\( FIS_{kp} \) = type \( p \) robot safety stock factor at DZ \( k \) (time units / planning horizon)

\( H \) = holding cost ($ / $ planning period)

\( L_{ij}^a \) = expected lead time for delivering raw material from supplier \( i \) to plant \( j \)

\( L_{jk}^p \) = expected lead time for delivering type \( p \) robots from plant \( j \) to DZ \( k \) (time / raw material)

### Costs:

\( C_i^a \) = procurement cost of raw material \( a \) provided by supplier \( i \) (including transportation and duties) ($ / robot)

\( C_k^p \) = distribution cost of type \( p \) robots employed at DZ \( k \) (including transportation and duties) ($ / robot)

\( C_{jk}^p \) = cost of type \( p \) robot distribution from plant \( j \) to DZ \( k \) ($ / period)

\( C_{kt}^p \) = production cost of type \( p \) robots used in DZ \( k \) in operation scale \( e \) during period \( t \) ($ / robot)
Expression for maximum number of periodic distribution trips per fleet at DZ:

\[ \eta_k \leq \text{MIN}\{z_{k}, HE_{k}, ZMAX_{k}, h_{b}\} \quad k \in K, \quad t \in T \]  

(4.3)

- The procurement reliability constraint (3.1) is in turn replaced by the following equivalent:

\[ \sum_{i \in G(i)} \sum_{j \in G(j)} v_{ij}^{k} \ln(\text{PROB}_{i}^{k}) \leq \ln(\text{PRO})_{i}^{k} \quad j \in J, p \in PJ(j) \]  

(4.4)

In order to deal with the non-linearity associated to the product of the continuum and binary variables, the approximation suggested by Hanson and Martin (1990) is applied, including variable redefinition and incorporating additional constraints. The procedure is carried out as follows:

Assuming that product \( \Omega \times \Xi \) appears in the model, where \( \Xi \) is a binary variable \( \{0,1\} \), while \( \Omega \) is a continuous non-negative variable, then the following procedure can be carried out:

\[ \Lambda \leq \Omega \]  

(4.5)

\[ \Delta \leq \Xi \Omega M_{\Omega} \]  

\[ \Lambda \geq \Omega - (1 - \Xi) M_{\Omega} \]

Where:

- \( M_{\Omega} \): maximum bound of \( \Omega \), corresponding to a positive integer parameter
- \( M_{\Xi} \): maximum bound of product \( \Omega \times \Xi \), corresponding to a positive integer parameter

The transformation can be used to approximate the non-linearity of equations (3.20) and (3.4). As a consequence, the following variables are defined and substituted in the aforementioned constraints:

\[ y_{k}^{p} = v_{k}^{p} w_{k}^{p} \quad k \in K, t \in T, p \in PK(k), e \in E(p) \]  

(4.6)

\[ x_{jk}^{p} = B_{jk} d_{jk}^{p} \quad j \in J, k \in K, p \in PJ(k,j) \]  

(4.7)

Finally, the following constraints are incorporated to the suggested model

\[ y_{k}^{p} \leq v_{k}^{p} \]  

(4.8)

\[ y_{k}^{p} = \text{GMAX}_{k}^{p} w_{k}^{p} \quad k \in K, t \in T, p \in PK(k), e \in E(p) \]  

(4.9)

\[ y_{k}^{p} \leq y_{k}^{p} \quad k \in K, t \in T, p \in PK(k), e \in E(p) \]  

(4.10)

\[ x_{jk}^{p} \leq d_{jk}^{p} \quad j \in J, k \in K, p \in PJ(k,j) \]  

(4.11)

\[ x_{jk}^{p} \leq (1 - B_{jk}) d_{jk}^{p} \quad j \in J, k \in K, p \in PJ(k,j) \]  

(4.12)

\[ x_{jk}^{p} \leq d_{jk}^{p} - (1 - B_{jk}) M \quad j \in J, k \in K, p \in PJ(k,j) \]  

(4.13)

- The stochastic demand constraint (3.18) is replaced by the expression below (the corresponding procedure is
presented in the Appendix 1). In consequence, the stochastic area in the mentioned equation is simplified into a Gaussian distribution, which is a commonly encountered condition in practical cases

\begin{equation}
\sum_{j=1}^{n} \sum_{\xi} \left[ \sum_{\phi} \phi \cdot \xi \cdot \psi_{j} - \sum_{\phi} \phi \cdot \xi \cdot \psi_{j} \right] = z_{\theta} \cdot e^{T} + E(\theta, \xi) \quad k \in K
\end{equation}

(4.14)

4.2. Step 2: Acquisition of valid constraints

The minimum and maximum objective bounds of the supply chain are obtained by means of a procedure that stipulates a single production scale range, containing two linear problems to be solved. In the first (second) one, the minimum (maximum) bound, is obtained by determining the maximum (minimum) scale cost for each factory-robot-period combination.

For both models, the associated procedure goes as follows: in the first place, each DZ’s product flow bounds are established:

\begin{equation}
GMAX_{k}^{P} = \max_{p \in PK(k)} \{GMAX_{k}^{P} \} \quad k \in K, p \in PK(k)
\end{equation}

(4.15)

\begin{equation}
GMIN_{k}^{P} = \min_{p \in PK(k)} \{GMIN_{k}^{P} \} \quad k \in K, p \in PK(k)
\end{equation}

(4.16)

Flow costs are determined as shown below:

- Model 1:

\begin{equation}
C_{k}^{P} = \max_{p \in PK(k)} \{C_{k}^{P} \} \quad k \in K, t \in T, p \in PK(k)
\end{equation}

(4.17)

- Model 2:

\begin{equation}
C_{k}^{P} = \min_{p \in PK(k)} \{C_{k}^{P} \} \quad k \in K, t \in T, p \in PK(k)
\end{equation}

(4.18)

In this step, equations (4.4), (3.20) and (3.11) are modified, and equations (3.9) and (3.10) are eliminated. The modified constraints are shown next:

\begin{equation}
\begin{aligned}
&\text{MIN} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{H} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{v} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{v} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad \text{subject to:}
\end{aligned}
\end{equation}

(4.19)

In this objective function the production and maintenance costs were customized.

Expression for stochastic demand constraint:

\begin{equation}
\sum_{j=1}^{n} \sum_{\xi} \frac{K_{j}}{\phi} \cdot \psi_{j} \cdot \xi = z_{\theta} \cdot e^{T} + E(\theta, \xi) \quad k \in K
\end{equation}

(4.20)

Expression for robot flow bounds at a given DZ:

\begin{equation}
GMIN_{k}^{P} \leq y_{k}^{P} \leq GMAX_{k}^{P} \quad k \in K, t \in T, p \in PK(k)
\end{equation}

(4.21)

Once the problems are solved, the objective function bounds can be determined.

Minimum bound – OFMIN : Min \{Model 1\}

Maximum bound – OFMAX : Min \{Model 2\}

The valid constraint obtained is incorporated to the original model and presented below:

\begin{equation}
\begin{aligned}
&\text{OFMIN} = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{H} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{v} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{v} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad \text{subject to:}
\end{aligned}
\end{equation}

(4.22)

Summarizing, it can be said that, as a result of the solution procedure, an MIP (and therefore a more treatable) formulation of the original problem is attained. Such formulation is made up of equations: 3.2, 3.3, 3.5 to 3.14, 3.16, 3.17, 3.19, 4.2 to 4.4, 4.8 to 4.14, and equation 4.22, together with its associated objective function.

5. Solution Procedure 2

If the production scale constraints (eq. 3.9, 3.10 and 3.11) are not directly included in the MNMIP model, but are contemplated later, once the new relaxed MIP problem has been solved, a new solution procedure can be obtained, as presented next.

5.1. Step 1: solving the relaxed MIP problem: the solution of the problem is expressed by constraints (equations) 3.2, 3.3, 3.5 to 3.8, 3.12 to 3.14, 3.16, 3.17, 3.19, 4.2 to 4.4, 4.11 to 4.14, 4.20, 4.21, and a modified objective function (eq. 4.19) from which the production cost has been removed, as shown below:

\begin{equation}
\begin{aligned}
&\text{MIN} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{H} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{v} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{v} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{V} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{l=1}^{L} C_{i}^{H} S_{j}^{D} \\
&\quad \text{subject to:}
\end{aligned}
\end{equation}

(5.1)

5.2. Step 2: integrating the production cost: In order to integrate the production cost into the model, the production scale constraints must be included. This allows to incorporate the algorithm below, starting from the optimal values of \( y_{k}^{P} \) obtained in the first step, and noted here as \( \psi_{k}^{P} \)
Begin

\[ S \leftarrow 0 \]

For \( k = 1 \) to \( N(k) \)

For \( t = 1 \) to \( N(t) \)

For \( p = 1 \) to \( N(p) \)

For \( e = 1 \) to \( N(e) \)

If \( \Phi_{kt}^p > 0 \) and \( GMIN_{kt}^p \leq \Phi_{kt}^p \leq GMAX_{kt}^p \)

Then \( \mathcal{Z} \leftarrow \{ k, t, p, e \} \)

\[ S \leftarrow S + c_{kt}^p \Phi_{kt}^p \]

End If

End For

End For

End For

End For

End

Where \( \mathcal{Z} \) is the set of indexes associated to the positive optimal flows \( \Phi_{kt}^p \), and \( S \) is the supply chain production cost. Finally, the production cost is added to the optimal solution of the objective function (eq. 4.19).

6. Sensibility Analysis

The sensibility analysis was performed using solution procedure 1, with the aid of a commercial LINGO 10™ software package. The computing of the scenarios was carried out on a Pentium-4 2.8 Ghz, 1GB RAM equipment, applying a Win XP-SP2 operational program. The problem comprises 20 suppliers handling 2 components each, 10 possible plant locations, 2 products in two production scales, and 20 Gaussian DZs. All possible combinations of logistic procurement connections were admitted, together with a distribution network featuring three DCs per production plant. Finally, a 5% Gaussian significance was used for the demand chance constraint. The used data can be seen in the Appendix 2. These problems have 14048 constraints and 4171 variables, of which 1241 are binary. The parameters under scrutiny were: area size, helicopter capacity and robot speed performance. Instance solutions take 500 seconds in average, with a maximum of 900 seconds. All the program runs were conducted with a maximum gap of 0.001, and most of the results were probably optimal.

The following table presents the percentage ranking of the average unitary costs that are part of the objective function:

<table>
<thead>
<tr>
<th>CHl</th>
<th>CF</th>
<th>C(_{\text{opt}})</th>
<th>C(_{\alpha})</th>
<th>C(_{\beta})</th>
<th>C(_{\alpha\beta})</th>
<th>CM</th>
<th>CC(_{\alpha})</th>
<th>CC(_{\beta})</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>35.62</td>
<td>0.0725</td>
<td>0.0534</td>
<td>0.0312</td>
<td>0.0044</td>
<td>0.04025</td>
<td>0.05</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

As it can be observed, the highest average unitary costs correspond to the strategic aspects, which are infrastructure distribution, and plant and distribution centre installation.

The sensibility analysis is applied by modifying the parameter percentage values and monitoring their impacts on the total cost of the supply chain.

6.1. Demand effect on minimum cost

Although the costs rise in direct relation to the expansion of the demand area, the non-linear nature of the problem becomes evident when changes in production scale and distribution infrastructure occur (e.g. more helicopters are required in order to satisfy a greater demand for robots). The smooth behaviour of the costs in some sections of Figure 1 is due to tactical changes in the supply chain. The slope remains unchanged as long as no adjustments in production or distribution infrastructure are undertaken in order to satisfy the demand, as can be observed in Figure 1.

FIGURE 1. Effect of demand on minimum cost

6.2. Effect of robot scanning speed on minimum cost

FIGURE 2. Effect of robot scanning speed on minimum cost
Robot scanning speed appears to be the most influential cost factor, since it constitutes the link between inventory, production and distribution activities in both strategic and tactical contexts. On one hand, the costs associated to production and distribution, diminish as scanning speed increases. On the other hand, inventory costs rise when there are an increasing number of unused robots during long time periods, as a consequence of the scanning speed increase. These two tendencies determine a general behaviour, according to which there is an overall cost percentage drop, marginally decreasing as the scanning speed is raised, due to the existence of counter-balance costs within the same objective function. (See figure 2).

6.3. Effect of helicopter capacity on minimum cost

Due to the fact that helicopters offer multiple usage possibilities, it is important to analyze their potential payload percentage per trip. As shown in figure 4, helicopter payload capacity has a significant impact on distribution costs, and therefore, on overall costs. Variation in total costs as a result of changes in helicopter payload capacity is similar to that shown in figure 2, resulting from robot scanning speed, as far as both parameters have a similar effect on distribution costs at DZs.

FIGURE 3. Effect of helicopter capacity on minimum cost

Additionally, the models detail optimal helicopter operation by considering each period’s trip frequency during the planning horizon.

For their solution, two procedures are suggested, allowing converting the original (and relatively hard) MINLP into a not so hard MIP. The efficiency of the second procedure tends to improve when the operation scales are considerable enough; otherwise, it lessens. On the other hand, the first procedure is advantageously explicit, in contrast with the second one, where the production cost is algorithmically induced.

Finally, the impact of parameter percentage variations on the overall cost is analyzed. The costs do not substantially differ if percentage variations do not entail changes in plant operation level or in the number of helicopters used in the distribution. The suggested solution procedure is considered satisfactory in terms of time for the analyzed example.

Among the new research possibilities opened up by this paper are the development of new solution procedures (e.g., decomposition methods) that allow the application of the model in larger scales of optimization, and the consideration of qualitative aspects that can be relevant in these types of organizations, such as transaction costs. The transaction costs are important because have a significant impact on the total costs, nevertheless these cannot be measured directly and require of new developments (like to IAM; (García et al., 2009) and so, can be included in the optimization of the supply chain.

Acknowledgments

The development of this work would have not been possible without the sponsorship of the Departamento de Ciencia y Tecnología de Colombia-COLCIENCIAS, the feedback from anonymous reviewers and the collaboration of our team, leaded by Carlos Alberto Vega Mejía.

Conclusions

The current paper presents an MIP and an MNMIP models for anti-personal landmine disposal by means of robots that—following the recommendations found in the cited references—include and integrate strategic and tactical relevant aspects of the supply chain. For that reason the model includes certain considerations, such as uncertain procurement, stochastic inventories in plants, production scales, supply-production-distribution capacities, particular distribution-production infrastructure, location-allocation considerations, stochastic demand, and BOM. Additionally, the models detail optimal helicopter operation by considering each period’s trip frequency during the planning horizon.

For their solution, two procedures are suggested, allowing converting the original (and relatively hard) MINLP into a not so hard MIP. The efficiency of the second procedure tends to improve when the operation scales are considerable enough; otherwise, it lessens. On the other hand, the first procedure is advantageously explicit, in contrast with the second one, where the production cost is algorithmically induced.

Finally, the impact of parameter percentage variations on the overall cost is analyzed. The costs do not substantially differ if percentage variations do not entail changes in plant operation level or in the number of helicopters used in the distribution. The suggested solution procedure is considered satisfactory in terms of time for the analyzed example.

Among the new research possibilities opened up by this paper are the development of new solution procedures (e.g., decomposition methods) that allow the application of the model in larger scales of optimization, and the consideration of qualitative aspects that can be relevant in these types of organizations, such as transaction costs. The transaction costs are important because have a significant impact on the total costs, nevertheless these cannot be measured directly and require of new developments (like to IAM; (García et al., 2009) and so, can be included in the optimization of the supply chain.

Acknowledgments

The development of this work would have not been possible without the sponsorship of the Departamento de Ciencia y Tecnología de Colombia-COLCIENCIAS, the feedback from anonymous reviewers and the collaboration of our team, leaded by Carlos Alberto Vega Mejía.

References


### Appendix 1

The chance approximation of the stochastic demand constraint is presented in the following equation:

\[
P \left[ \sum_{k \in K} \sum_{i \in N} \left( \sum_{j \in G_k} \left( \sum_{l \in L_j} \left( \sum_{t \in T_{j,l}} \left( \sum_{p \in P} \phi^p \left( x^p \right)^j t^l + \sum_{p \in P} \phi^p \left( x^p - 1 \right)^j t^l w_{j,l}^p \right) \right) \right) \right) \right] \leq \Theta(\xi) \geq 1 - \alpha \tag{A1}
\]

\[
P \left[ \sum_{k \in K} \sum_{i \in N} \left( \sum_{j \in G_k} \left( \sum_{l \in L_j} \left( \sum_{t \in T_{j,l}} \left( \sum_{p \in P} \phi^p \left( x^p \right)^j t^l + \sum_{p \in P} \phi^p \left( x^p - 1 \right)^j t^l w_{j,l}^p \right) \right) \right) \right) \right] - E(\Theta(\xi)) \leq z_{1-\alpha} \tag{A2}
\]

\[
P \left[ \sum_{k \in K} \sum_{i \in N} \left( \sum_{j \in G_k} \left( \sum_{l \in L_j} \left( \sum_{t \in T_{j,l}} \left( \sum_{p \in P} \phi^p \left( x^p \right)^j t^l + \sum_{p \in P} \phi^p \left( x^p - 1 \right)^j t^l w_{j,l}^p \right) \right) \right) \right) \right] - E(\Theta(\xi)) \geq z_{1-\alpha} \tag{A3}
\]

Under the normal demand assumption

\[
\sum_{k \in K} \sum_{i \in N} \left( \sum_{j \in G_k} \left( \sum_{l \in L_j} \left( \sum_{t \in T_{j,l}} \left( \sum_{p \in P} \phi^p \left( x^p \right)^j t^l + \sum_{p \in P} \phi^p \left( x^p - 1 \right)^j t^l w_{j,l}^p \right) \right) \right) \right) \right] - E(\Theta(\xi)) \geq z_{1-\alpha} \tag{A4}
\]
\[ \sum_{p \in P \cap C(i)} \sum_{k \in K} \left[ \sum_{l=1}^{K} \theta_p^p \Phi_p^p \pi_p^p y_{k,l}^j w_{k,l}^j + \sum_{l=0}^{K-1} \theta_p^p \Phi_p^p (\pi_p^p - 1) y_{k,\pi_p^p+1,l}^p + w_{k,\pi_p^p+1,l}^p \right] + z_{i-k} a(\Theta_k(\xi)) + E(i) \]  

(A5)

Appendix 2

DCs (k): 20  
Plants (j): 10  
Raw materials (a): 2  
Scales (e): 2  
Suppliers (i): 20  
Time periods (t): 10  
Types of robots (p): 2

TABLE 2.1. Values of parameter \( \text{PROBJ}_i^p \)

<table>
<thead>
<tr>
<th>i</th>
<th>j=1</th>
<th>j=2</th>
<th>j=3</th>
<th>j=4</th>
<th>j=5</th>
<th>j=6</th>
<th>j=7</th>
<th>j=8</th>
<th>j=9</th>
<th>j=10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a=1</td>
<td>a=2</td>
<td>a=1</td>
<td>a=2</td>
<td>a=1</td>
<td>a=2</td>
<td>a=1</td>
<td>a=2</td>
<td>a=1</td>
<td>a=2</td>
</tr>
<tr>
<td>1</td>
<td>0.98</td>
<td>0.95</td>
<td>0.92</td>
<td>0.92</td>
<td>0.95</td>
<td>0.95</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>2</td>
<td>0.95</td>
<td>0.94</td>
<td>0.98</td>
<td>0.98</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>3</td>
<td>0.94</td>
<td>0.93</td>
<td>0.99</td>
<td>0.99</td>
<td>0.94</td>
<td>0.96</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>4</td>
<td>0.93</td>
<td>0.92</td>
<td>0.96</td>
<td>0.96</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>0.92</td>
<td>0.98</td>
<td>0.95</td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>6</td>
<td>0.98</td>
<td>0.99</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>7</td>
<td>0.99</td>
<td>0.96</td>
<td>0.91</td>
<td>0.91</td>
<td>0.96</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>8</td>
<td>0.96</td>
<td>0.98</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>9</td>
<td>0.95</td>
<td>0.99</td>
<td>0.93</td>
<td>0.93</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>10</td>
<td>0.95</td>
<td>0.96</td>
<td>0.92</td>
<td>0.92</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>11</td>
<td>0.91</td>
<td>0.95</td>
<td>0.98</td>
<td>0.98</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>12</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>13</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>14</td>
<td>0.96</td>
<td>0.94</td>
<td>0.96</td>
<td>0.96</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>15</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.96</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>16</td>
<td>0.98</td>
<td>0.96</td>
<td>0.96</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>17</td>
<td>0.94</td>
<td>0.95</td>
<td>0.91</td>
<td>0.91</td>
<td>0.98</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>18</td>
<td>0.95</td>
<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>19</td>
<td>0.96</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>20</td>
<td>0.95</td>
<td>1</td>
<td>0.96</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

TABLE 2.2. Values of parameters \( PROJ_i^p \), \( CAPJ_i^p \), \( CF_i \)

<table>
<thead>
<tr>
<th></th>
<th>( PROJ_i^p )</th>
<th>( CAPJ_i^p )</th>
<th>( CF_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p=1 )</td>
<td>( p=2 )</td>
<td>( p=1 )</td>
</tr>
<tr>
<td>( j=1 )</td>
<td>0.95</td>
<td>0.96</td>
<td>0.94</td>
</tr>
<tr>
<td>( j=2 )</td>
<td>0.95</td>
<td>0.96</td>
<td>0.94</td>
</tr>
<tr>
<td>( j=3 )</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( j=4 )</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.
### Table 2.3. Values of parameters $Q^p, O^p, \beta^p, \theta^p, \psi^p, \pi^p, CM$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$Q^p$</th>
<th>$O^p$</th>
<th>$\beta^p$</th>
<th>$\theta^p$</th>
<th>$\psi^p$</th>
<th>$\pi^p$</th>
<th>$CM$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>0.0001</td>
<td>0.1</td>
<td>4</td>
</tr>
<tr>
<td>a=2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>0.0002</td>
<td>0.12</td>
<td>5</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

### Table 2.4. Values of parameters $CI^p, FIS^p$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$FIS^p$</th>
<th>$CI^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.28571429</td>
<td>0.76190476</td>
</tr>
<tr>
<td>2</td>
<td>0.31428571</td>
<td>0.76190476</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

### Table 2.5. Values of parameters $SMIN^a, SMAX^a$

<table>
<thead>
<tr>
<th>$a$</th>
<th>$SMIN^a$</th>
<th>$SMAX^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

### Table 2.6. Values of parameters $GMIN^a, GMAX^a, ZMAX^a, HE^a, E(\Theta_0^a), FIS^p$

<table>
<thead>
<tr>
<th>$a$</th>
<th>$GMIN^a$</th>
<th>$GMAX^a$</th>
<th>$ZMAX^a$</th>
<th>$HE^a$</th>
<th>$E(\Theta_0^a)$</th>
<th>$FIS^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.28571429</td>
<td>0.76190476</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.31428571</td>
<td>0.76190476</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

### Table 2.7. Values of parameter $XMAX^a$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$XMAX^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

---

**Revisión Innovar Vol. 22, Núm. 45, Julio-Septiembre de 2012**  
65
### Investigación colombiana

TABLA 2.8. Valores de parámetro $L_j$

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11</td>
<td>0.69</td>
<td>0.9</td>
<td>0.05</td>
<td>0.67</td>
<td>0.03</td>
<td>0.26</td>
<td>0.75</td>
<td>0.95</td>
<td>0.16</td>
</tr>
<tr>
<td>2</td>
<td>0.33</td>
<td>0.61</td>
<td>0.49</td>
<td>0.71</td>
<td>0.11</td>
<td>0.9</td>
<td>0.35</td>
<td>0.21</td>
<td>0.3</td>
<td>0.72</td>
</tr>
<tr>
<td>3</td>
<td>0.88</td>
<td>0.93</td>
<td>0.98</td>
<td>0.2</td>
<td>0.06</td>
<td>0.25</td>
<td>0.46</td>
<td>0.12</td>
<td>0.5</td>
<td>0.93</td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>0.87</td>
<td>0.85</td>
<td>0.07</td>
<td>0.44</td>
<td>0.71</td>
<td>0.97</td>
<td>0.86</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>5</td>
<td>0.09</td>
<td>0.22</td>
<td>0.17</td>
<td>0.06</td>
<td>0.33</td>
<td>0.91</td>
<td>0.46</td>
<td>0.17</td>
<td>0.32</td>
<td>0.68</td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
<td>0.15</td>
<td>0.6</td>
<td>0.6</td>
<td>0.99</td>
<td>0.96</td>
<td>0.35</td>
<td>1</td>
<td>0.95</td>
<td>0.87</td>
</tr>
<tr>
<td>7</td>
<td>0.21</td>
<td>0.12</td>
<td>0.02</td>
<td>0.68</td>
<td>0.76</td>
<td>0.86</td>
<td>0.62</td>
<td>0.29</td>
<td>0.67</td>
<td>0.84</td>
</tr>
<tr>
<td>8</td>
<td>0.54</td>
<td>0.87</td>
<td>0.89</td>
<td>0.46</td>
<td>0.27</td>
<td>0.82</td>
<td>0.44</td>
<td>0.98</td>
<td>0.89</td>
<td>0.11</td>
</tr>
<tr>
<td>9</td>
<td>0.43</td>
<td>0.32</td>
<td>0.58</td>
<td>0.52</td>
<td>0.13</td>
<td>0.39</td>
<td>0.77</td>
<td>0.49</td>
<td>0.3</td>
<td>0.99</td>
</tr>
<tr>
<td>10</td>
<td>0.01</td>
<td>0.8</td>
<td>0.72</td>
<td>0.52</td>
<td>0.17</td>
<td>0.76</td>
<td>0.17</td>
<td>0.76</td>
<td>0.45</td>
<td>0.57</td>
</tr>
<tr>
<td>11</td>
<td>0.55</td>
<td>0.17</td>
<td>0.7</td>
<td>0.02</td>
<td>0.1</td>
<td>0.27</td>
<td>0.59</td>
<td>0.52</td>
<td>0.63</td>
<td>0.58</td>
</tr>
<tr>
<td>12</td>
<td>0.87</td>
<td>0.24</td>
<td>1</td>
<td>0.44</td>
<td>0.35</td>
<td>0.38</td>
<td>0.86</td>
<td>0.78</td>
<td>0.53</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

TABLA 2.9. Valores de parámetro $F_j$

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
<td>0.08</td>
<td>0.11</td>
<td>0.94</td>
<td>0.02</td>
<td>0.13</td>
<td>0.8</td>
<td>0.56</td>
<td>0.38</td>
<td>0.76</td>
</tr>
<tr>
<td>2</td>
<td>0.16</td>
<td>0.66</td>
<td>0.19</td>
<td>0.02</td>
<td>0.62</td>
<td>0.78</td>
<td>0.9</td>
<td>0.08</td>
<td>0.42</td>
<td>0.89</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>0.16</td>
<td>0.13</td>
<td>0.05</td>
<td>0.09</td>
<td>0.95</td>
<td>0.63</td>
<td>0.52</td>
<td>0.71</td>
<td>0.88</td>
</tr>
<tr>
<td>4</td>
<td>0.38</td>
<td>0.33</td>
<td>0.58</td>
<td>0.22</td>
<td>0.88</td>
<td>0.43</td>
<td>0.13</td>
<td>0.14</td>
<td>0.26</td>
<td>0.08</td>
</tr>
<tr>
<td>5</td>
<td>0.69</td>
<td>0.56</td>
<td>0.92</td>
<td>0.39</td>
<td>0.82</td>
<td>0.27</td>
<td>0.51</td>
<td>0.92</td>
<td>0.5</td>
<td>0.97</td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>0.44</td>
<td>0.31</td>
<td>0.09</td>
<td>0.42</td>
<td>0.01</td>
<td>0.18</td>
<td>0.53</td>
<td>0.27</td>
<td>0.98</td>
</tr>
<tr>
<td>7</td>
<td>0.11</td>
<td>0.11</td>
<td>0.23</td>
<td>0.98</td>
<td>0.19</td>
<td>0.85</td>
<td>0.26</td>
<td>0.29</td>
<td>0.82</td>
<td>0.32</td>
</tr>
<tr>
<td>8</td>
<td>0.72</td>
<td>0.74</td>
<td>0.48</td>
<td>0.68</td>
<td>0.86</td>
<td>0.15</td>
<td>0.12</td>
<td>0.68</td>
<td>0.62</td>
<td>0.94</td>
</tr>
<tr>
<td>9</td>
<td>0.23</td>
<td>0.37</td>
<td>0.83</td>
<td>0.54</td>
<td>0.12</td>
<td>0.17</td>
<td>0.83</td>
<td>0.24</td>
<td>0.52</td>
<td>0.17</td>
</tr>
<tr>
<td>10</td>
<td>0.05</td>
<td>0.4</td>
<td>0.14</td>
<td>0.94</td>
<td>0.15</td>
<td>0.29</td>
<td>0.51</td>
<td>0.92</td>
<td>0.11</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

TABLA 2.10. Valores de parámetro $P_j$

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11</td>
<td>0.69</td>
<td>0.9</td>
<td>0.05</td>
<td>0.67</td>
<td>0.03</td>
<td>0.26</td>
<td>0.75</td>
<td>0.95</td>
<td>0.16</td>
</tr>
<tr>
<td>2</td>
<td>0.33</td>
<td>0.61</td>
<td>0.49</td>
<td>0.71</td>
<td>0.11</td>
<td>0.9</td>
<td>0.35</td>
<td>0.21</td>
<td>0.3</td>
<td>0.72</td>
</tr>
<tr>
<td>3</td>
<td>0.88</td>
<td>0.93</td>
<td>0.98</td>
<td>0.2</td>
<td>0.06</td>
<td>0.25</td>
<td>0.46</td>
<td>0.12</td>
<td>0.5</td>
<td>0.93</td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>0.87</td>
<td>0.85</td>
<td>0.07</td>
<td>0.44</td>
<td>0.71</td>
<td>0.97</td>
<td>0.86</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>5</td>
<td>0.09</td>
<td>0.22</td>
<td>0.17</td>
<td>0.06</td>
<td>0.33</td>
<td>0.91</td>
<td>0.46</td>
<td>0.17</td>
<td>0.32</td>
<td>0.68</td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
<td>0.15</td>
<td>0.6</td>
<td>0.6</td>
<td>0.99</td>
<td>0.96</td>
<td>0.35</td>
<td>1</td>
<td>0.95</td>
<td>0.87</td>
</tr>
<tr>
<td>7</td>
<td>0.21</td>
<td>0.12</td>
<td>0.02</td>
<td>0.68</td>
<td>0.76</td>
<td>0.86</td>
<td>0.62</td>
<td>0.29</td>
<td>0.67</td>
<td>0.84</td>
</tr>
<tr>
<td>8</td>
<td>0.54</td>
<td>0.87</td>
<td>0.89</td>
<td>0.46</td>
<td>0.27</td>
<td>0.82</td>
<td>0.44</td>
<td>0.98</td>
<td>0.89</td>
<td>0.11</td>
</tr>
<tr>
<td>9</td>
<td>0.43</td>
<td>0.32</td>
<td>0.58</td>
<td>0.52</td>
<td>0.13</td>
<td>0.39</td>
<td>0.77</td>
<td>0.49</td>
<td>0.3</td>
<td>0.99</td>
</tr>
<tr>
<td>10</td>
<td>0.09</td>
<td>0.22</td>
<td>0.17</td>
<td>0.06</td>
<td>0.33</td>
<td>0.91</td>
<td>0.46</td>
<td>0.17</td>
<td>0.32</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.
### Table 2.11. Values of parameters \( CC_{kl}, CH_{kl} \)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CC_{kl} )</td>
<td>1610300</td>
<td>1659300</td>
<td>1809200</td>
<td>1898900</td>
<td>2012800</td>
<td>2191600</td>
<td>2303100</td>
<td>2480100</td>
<td>2583600</td>
<td>2354100</td>
</tr>
<tr>
<td>( CH_{kl} )</td>
<td>1610300</td>
<td>1659300</td>
<td>1809200</td>
<td>1898900</td>
<td>2012800</td>
<td>2191600</td>
<td>2303100</td>
<td>2480100</td>
<td>2583600</td>
<td>2354100</td>
</tr>
</tbody>
</table>

### Table 2.12. Values of parameter \( C_{ij} \)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{ij} )</td>
<td>1610300</td>
<td>1659300</td>
<td>1809200</td>
<td>1898900</td>
<td>2012800</td>
<td>2191600</td>
<td>2303100</td>
<td>2480100</td>
<td>2583600</td>
<td>2354100</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

### Table 2.13. Values of parameter \( C_{ik} \)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{ik} )</td>
<td>1610300</td>
<td>1659300</td>
<td>1809200</td>
<td>1898900</td>
<td>2012800</td>
<td>2191600</td>
<td>2303100</td>
<td>2480100</td>
<td>2583600</td>
<td>2354100</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

REVISTA INNOVAR VOL. 22, NÚM. 45, JULIO-SEPTIEMBRE DE 2012
### TABLE 2.13. Values of parameter $F_{jk}$

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
<td>0.1</td>
<td>0.39</td>
<td>0.38</td>
<td>0.69</td>
<td>0.9</td>
<td>0.11</td>
<td>0.72</td>
<td>0.21</td>
<td>0.05</td>
<td>0.82</td>
<td>0.14</td>
<td>0.94</td>
<td>0.22</td>
<td>0.45</td>
<td>0.39</td>
<td>0.89</td>
<td>0.29</td>
<td>0.09</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>0.11</td>
<td>0.19</td>
<td>0.3</td>
<td>0.58</td>
<td>0.92</td>
<td>0.31</td>
<td>0.23</td>
<td>0.48</td>
<td>0.83</td>
<td>0.14</td>
<td>0.44</td>
<td>0.13</td>
<td>0.06</td>
<td>0.29</td>
<td>0.18</td>
<td>0.65</td>
<td>0.81</td>
<td>0.44</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.02</td>
<td>0.62</td>
<td>0.09</td>
<td>0.88</td>
<td>0.82</td>
<td>0.42</td>
<td>0.19</td>
<td>0.86</td>
<td>0.12</td>
<td>0.15</td>
<td>0.32</td>
<td>0.69</td>
<td>0.13</td>
<td>0.12</td>
<td>0.56</td>
<td>0.29</td>
<td>0.05</td>
<td>0.42</td>
<td>0.74</td>
<td>0.67</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
<td>0.9</td>
<td>0.66</td>
<td>0.13</td>
<td>0.51</td>
<td>0.18</td>
<td>0.26</td>
<td>0.12</td>
<td>0.83</td>
<td>0.51</td>
<td>0.11</td>
<td>0.8</td>
<td>0.28</td>
<td>0.13</td>
<td>0.21</td>
<td>0.4</td>
<td>0.76</td>
<td>0.99</td>
<td>0.68</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
<td>0.47</td>
<td>0.73</td>
<td>0.26</td>
<td>0.5</td>
<td>0.27</td>
<td>0.82</td>
<td>0.62</td>
<td>0.52</td>
<td>0.11</td>
<td>0.37</td>
<td>0.34</td>
<td>0.52</td>
<td>0.79</td>
<td>0.84</td>
<td>0.48</td>
<td>0.75</td>
<td>0.38</td>
<td>0.52</td>
<td>0.95</td>
</tr>
<tr>
<td>6</td>
<td>0.88</td>
<td>0.8</td>
<td>0.53</td>
<td>0.3</td>
<td>0.9</td>
<td>0.08</td>
<td>0.45</td>
<td>0.95</td>
<td>0.35</td>
<td>0.75</td>
<td>0.37</td>
<td>0.73</td>
<td>0.77</td>
<td>0.85</td>
<td>0.32</td>
<td>0.75</td>
<td>0.88</td>
<td>0.17</td>
<td>0.99</td>
<td>0.89</td>
</tr>
<tr>
<td>7</td>
<td>0.88</td>
<td>0.69</td>
<td>0.88</td>
<td>0.6</td>
<td>0.93</td>
<td>0.3</td>
<td>0.03</td>
<td>0.96</td>
<td>0.5</td>
<td>0.66</td>
<td>0.28</td>
<td>0.97</td>
<td>0.26</td>
<td>0.95</td>
<td>0.63</td>
<td>0.51</td>
<td>0.68</td>
<td>0.67</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.76</td>
<td>0.68</td>
<td>0.88</td>
<td>0.24</td>
<td>0.89</td>
<td>0.78</td>
<td>0.56</td>
<td>0.87</td>
<td>0.86</td>
<td>0.6</td>
<td>0.81</td>
<td>0.88</td>
<td>0.58</td>
<td>0.76</td>
<td>0.1</td>
<td>0.01</td>
<td>0.45</td>
<td>0.94</td>
<td>0.7</td>
<td>0.37</td>
</tr>
<tr>
<td>9</td>
<td>0.95</td>
<td>0.85</td>
<td>0.74</td>
<td>0.92</td>
<td>0.81</td>
<td>0.07</td>
<td>0.27</td>
<td>0.91</td>
<td>0.98</td>
<td>0.75</td>
<td>0.37</td>
<td>0.64</td>
<td>0.02</td>
<td>0.61</td>
<td>0.61</td>
<td>0.7</td>
<td>0.32</td>
<td>0.47</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>10</td>
<td>0.04</td>
<td>0.32</td>
<td>0.5</td>
<td>0.34</td>
<td>0.25</td>
<td>0.42</td>
<td>0.53</td>
<td>0.48</td>
<td>0.98</td>
<td>0.93</td>
<td>0.21</td>
<td>0.12</td>
<td>0.56</td>
<td>0.39</td>
<td>0.09</td>
<td>0.56</td>
<td>0.53</td>
<td>0.5</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.

### TABLE 2.14. Set $RJK(j,k)$

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Fuente: elaboración propia.